Reply by Authors to D. J. Johns and S. Gopalacharyulu

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ALTHOUGH Bushnell's method[2] for buckling of shells of revolution under non-axisymmetric loads has no basis for comparison with the authors'[1] in the absence of theoretical grounding, the nature of the approximation may be analyzed with a view to rules for users of BOSOR 4. A study is in progress here accordingly, with emphasis on the cases in which the buckling loads according to Ref. 2 deviate considerably lower and higher than their counterparts according to Ref. 1 and the latter method is therefore justified.

The first category is exemplified[3] by an annular plate (inner radius $r = 3.8$, outer radius $R = 41.8$, thickness $t = 0.2$, Poisson's ratio $v = 0.3$, and Young's modulus $E = 2\times 10^6$) under a symmetric load, $N_\xi = N_{\xi_0} + N_{\xi_1} * \cos\theta$, at its outer circumference. Boundary conditions on buckling are: inner circumference $U = V = W = M_\xi = 0$; outer circumference $N_\xi = N_{\xi\theta} = W = M_\xi = 0$. Buckling loads λ were examined for different combinations of N_{ξ_1} and N_{ξ_0}. At $N_{\xi_0} = 0$, $N_{\xi_1} = -1$, the result was $\lambda = 13.58$ in Ref. 1 and $\lambda = 9.314$ in Ref. 2, a 31% difference, indicating the exact method.

In the second category, there are numerous cases in which the buckling patterns for axisymmetric and non-axisymmetric loads are completely unrelated, and according to Ref. 2, is quite likely to be higher than its counterpart of Ref. 1. Such a case is the same as the preceding example in which $N_{\xi_0} = -1$ and $N_{\xi_1} = -1$. The result of the eigenvalue λ $[N_{cr} = \lambda(N_{\xi_0} + N_{\xi_1})]$ was $\lambda = 7.135$ in Ref. 1 and $\lambda = 14.948$ in Ref. 2.

Another case, for example, is where buckling may occur in axial compression along a meridian, in a long cylinder under a symmetric load (as in Ref. 1), especially if the shell is orthotropic with a large difference between the circumferential and meridional stiffness. (By contrast, under axisymmetric hydrostatic pressure, buckling occurs in tangential compression.)

The buckling load in axisymmetric loading is the same whether determined from the symmetric or the antisymmetric mode; in non-axisymmetric cases with the buckling load differing substantially between the two modes, the solution according to Ref. 2, which is ultimately based on an equivalent axisymmetric prebuckling load, will deviate considerably from the exact one.

For symmetric loads, the buckling mode is either symmetric or antisymmetric, and the buckling load can be determined for each case in order to ascertain which of them yields the lower eigenvalue. This is extremely convenient from the computer time and memory viewpoints, in that instead of considering a single case involving all variables, the symmetric and antisymmetric variables are treated separately in their respective cases. The numerical problems in the authors' paper were worked out accordingly. Concerning the writers' recommendation that only "effective" terms in the Fourier series [Eq. (13) in Ref. 1] be considered, this is unfeasible under the authors' approach (for proof, see Ref. 4).

References

[1] Sheinman, I. and Tene, Y., "Buckling in segmented shells of revolution subjected to symmetric and antisymmetric loads," *AIAA Journal*, Vol. 12, No. 1, Jan. 1974, pp. 15–20.

[2] Bushnell, D., "Stress, Stability and Vibration of Complex Branched Shells of Revolution," *Analysis and Users' Manual for BOSOR 4*, CR-2116, Oct. 1972, NASA.

[3] Sheinman, I., "Buckling of Shells of Revolution Subjected to Non-Axisymmetric Loads," D.Sc. thesis, Technion, Israel, 1972 (in Hebrew).

[4] Sheinman, I., "Analysis of shells of revolution having arbitrary stiffness distribution, including shear deformation," M.Sc. thesis, Technion, Israel, 1970 (in Hebrew).

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Comment on "Mathematical Modeling of Spinning Elastic Bodies for Modal Analysis"

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THE derivation of the continuum equation for a radial beam rotating normal to the spin axis of its inertially rotating base as presented by Likins, Barbera, and Baddeley,[1] is not entirely satisfying. An alternate slightly generalized development which avoids the effective applied load concept is given below. The notation and coordinate system of Ref. 1 are employed.

As in Ref. 1, consider the radial beam as a solid one in which, under deformation, plane sections remain plane. The displacement at points within the continuum may be represented by

$$u(x, y, x, t) = \xi(y, t) \tag{1}$$

$$v(x, y, z, t) = \zeta(y, t) - x\xi_y(y, t) - z\eta_y(y, t) - \frac{1}{2}\int_0^y \{\xi_y{}^2(y', t) + \eta_y{}^2(y', t)\}\, dy' \tag{2}$$

$$w(x, y, z, t) = \eta(y, t) \tag{3}$$

In Eqs. (1) and (3), displacement is approximated to linear order in $\xi(x, t)$ and $\eta(x, t)$. The axial displacement, $v(x, y, z, t)$, is composed of three components: a) displacement associated with uniform elastic extension of the neutral line [the first term on the right-hand side of Eq. (2)]; b) displacement associated with fiber strain of "plane sections" bending [the second and third terms on the right-hand side of Eq. (2)]; c) displacement associated with the "foreshortening effect" (for derivation see, for example, Appendix B of Ref. 2). It is assumed that the point on the beam's neutral axis at the origin 0 undergoes no elastic displacement. Equation (2) is appropriate to quadratic order in ξ and η. Equations (1–3) are further predicated on the assumption that beam torsion is ignorable; the development may be readily generalized to account for torsion.

Substitution of Eqs. (1–3) into (12–17) of Ref. 1, yields

$$\varepsilon_{xx} = \varepsilon_{zz} = \varepsilon_{xy} = \varepsilon_{yz} = \varepsilon_{xz} = 0 \tag{4}$$

to a linear order approximation, and

$$\varepsilon_{yy} = (\zeta_y - x\xi_{yy} - z\eta_{yy}) + \tfrac{1}{2}(\zeta_y - x\xi_{yy} - z\eta_{yy})^2 \tag{5}$$

Substitution of Eqs. (4) and (5) into Eq. (11) of Ref. 1 yields, to quadratic order approximation

$$V = (E/2)\int_0^l (A\zeta_y{}^2 + I_z\xi_{yy}{}^2 + I_x\eta_{yy}{}^2)\, dy \tag{6}$$

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